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National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No.NAS-5-12487

ST —PF —GM —10752

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ON THE LONGITUDINAL ELECTRIC FIELD IN THE MAGNETOSPHERE AT QUASIMAXWELLIAN DISTRIBUTION OF CHARGED PARTICLES

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31 AUGUST 1968

ST-PF-GM-10752

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Kosmicheskiye Issledovaniya
Tom 6, vypusk 4, 604 - 610,
Izdatel'stvo "NAUKA", 1968.

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SUMMARY

The distribution of the longitudinal electric field and the density of charged particles are investigated in a one-dimensional symmetrical magnetic mirror for a given potential difference between the magnetic equator and a certain plane, behind which total absorption of charged particles takes place. It is assumed that the charged particles whose mirror points are located between the equator and the absorption plane, have a Maxwellian distribution by velocities, while particles with mirror points situated outside that region, are absent (such a distribution by velocities is called quasi-Maxwellian).

Discussed here is the formation mechanism of the soft component in the outer trapped radiation belt, which is linked with the escape of charged particles of low energy into the polar atmosphere under the action of a longitudinal electric field in combination with the azimuthal particle drift in a nonuniform magnetic field, and in the presence of directed motion of matter in the Earth's magnetosphere.

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The Earth's magnetosphere constitutes a region inaccessible for the plasma of solar corona (forbidden zone) [1]. It may be shown that in a quasi-hydrodynamic two-dimensional model of rarefied plasma supersonic flow in the equatorial plane of the magnetic dipole, there exist a motion of matter inside the Earth's magnetosphere (fountain effect in the forbidden zone) [2]. Thus, there is inside the magnetosphere an electric field perpendicular to magnetic lines of force. In reality, the motion of matter inside the magnetosphere must also result in the emergence of a longitudinal electric field; this is due to the absence of collisions and the inhomogeneity of the magnetic field (in this region of rarefied plasma the notion of longitudinal conductivity is inapplicable [3]). The study of the acceleration mechanism of aurora protons on rockets shows that this mechanism acts at altitudes of the order of 300 km, and that it has the following peculiarities:

1) at low altitudes a preferential motion of protons is noted, upward along the magnetic field lines;

2) the pitch-angles of protons, moving upward along the field lines, correspond to mirror points in the atmosphere;

3) a decrease is observed of the downward directed proton flux with mirror points lying above the atmosphere, whereupon the proton flux directed upward does not decrease.

These peculiarities of the acceleration mechanism may be satisfactorily explained if one assumes that protons of polar aurorae are accelerated by a longitudinal electric field at altitudes greater than 300 km directed upward along the magnetic field li es, while the corresponding difference in potentials attains tens of kilovolts [4]. The same conclusions follow from the observation in the aurora zone of red shift of hydrogen spectral lines in the direction of the magnetic zenith [5]. Finally, the same order of magnitude is given the electric field by direct measurements in the polar aurora zone [6]. This is why, it appears to be of interest to investigate, on the one part, the character and the causes of existence of longitudinal electric fields, and the study of the influence of these fields on the distribution of charged particles, the variation of particle distribution functions, the Doppler shift of emission spectra, etc., on the other.

Proposed in [7] is an acceleration mechanism of aurora electrons in the magnetosphere, linked with the excitation of ionic sound as a consequence of instability of the plasma, in which current flows along the magnetic field lines (electrodynamic turbulent heating of electrons). This mechanism induces shortlived longitudinal electric fields. The second possibility is the existence of longitudinal electrostatic fields; it is considered in [3, 8]. If the collision effect is negligibly small, the existence of a longitudinal electric field becomes a necessity in case of different distributions of electrons and ions by pitch-angles for assuring the quasineutrality.

Beyond the plasmapause, the density of the magnetospheric plasma is such that $r_d \ll L \ll \lambda$, where r_d is the Debye radius, L is the characteristic dimension of the problem and λ is the length of the free path of the charged particle. Because of the smallness of r_d, the longitudinal electric field must be determined from the condition of quasineutrality, $n_e = n_i$; if for certain given distribution functions of charged particles by velocities this condition can not be fulfilled, the corresponding stationary states are impossible [9]. The distribution of the electrostatic field and of the density of charged particles in a onedimensional symmetric mirror, for a given potential difference between the equator and a certain plane, above which particles are fully absorbed, is studied in [9]. It is assumed that all charged particles pass through the magnetic equatorial plane; theoremes are demonstrated of the existence and uniqueness of solutions for monoenergetic distributions and results are presented for monoenergetic quasi-isotropic distributions. Using the same model, the present work considers the more complex case of nonmonoenergetic, quasi-Maxwellian distributions of electrons and ions by velocities.

DENSITY OF CHARGED PARTICLES AND EQUATION OF QUASINEUTRALITY

Let us consider a magnetic field tube, unidimensional in the sense that the spatial dependence of all quantities is described by a single coordinate s. Assume that the magnetic field B has a minimum B_0 at s = 0 and increases monotonically on both sides of the plane s = 0, which we shall call the equator. Let us denote $\gamma = B/B_0$ (γ has its greatest value γ_l at $s = l$). We shall consider that the particles passing through the point $s = l$ abandon the considered system. We limit ourselves to the case of stationary electric and magnetic fields in the presence of rarefied plasma consisting of electrons and singly charged ions $(r_d \ll L \ll \lambda)$.

Let the electric field be directed from the equator and the difference be given of potentials between the points $s = l$ and s = 0. We shall assume that the electrostatic pot ntial at the equator is zero and denote all quantities related to the equator by index zero and those related to $s = l$, by index l. Inasmuch as the energy of the particle is an integral of motion and the magnetic moment is an adiabatic invariant, the particle motion is described by the equations

$$V + U = V_0 + U_0, \qquad V \sin^2 \varphi + \gamma V_0 \sin^2 \varphi_0, \tag{1}$$

where $\sin \varphi = v_\perp / v$. v and $v_\perp$ are respectively the velocity modulus and its transverse component, V is the kinetic and U is the potential energy of the particle.

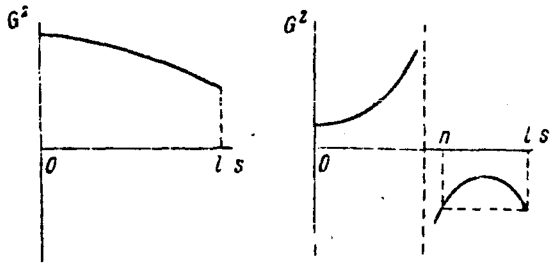

Fig.1

In these denotations the Maxwellian distribution function has the form

$$f = f_0 \exp\left(-\frac{V + U}{2T}\right), \tag{2}$$

where f_0 is the dimensional coefficient and T is the temperature.

The equation (1) of particle motion may be represented in the form

$$G^2 \sin^2 \varphi = C, \tag{3}$$

where C is a constant,

$$G^2 = \frac{V}{\gamma(V+U)}. \tag{4}$$

The dependence of G^2 on s in the case when the electric potential is a monotonic function of s is plotted in Fig.1.

Let us assume that the particles have a quasi-Maxwellian distribution by velocities; this means that the particles, whose mirror points lie between the equator and the plane $s=l$, have a Maxwellian distribution by velocities (2), and that there are no particles with mirror points above $s=l$. At the given point the density is represented by the integral from the distribution function by velocities

$$n = 4\pi f_0 \int_0^\infty \exp\left(-\frac{V+U}{2T}\right) v^2\, dv \int_\mu \sin\varphi\, d\varphi, \tag{5}$$

where μ is the limit of the integration region by pitch-angles for a given energy V of the particle, determined by the above indicated method by the position of particle's mirror points. It is easy to see that for the chosen direction of the electric field for ions of sufficiently high energies and electrons of any energies, the limits of integration by pitch-angles are arc sin G_l/G) and $\pi/2$ (Fig.1, to the left), for the least value of G is reached at the point $s=l$ [9]; these are polar-equatorial particles [10]. Consequently, the density of electrons may be represented in the form

$$n_e = 2\pi f_{0e}\left(\frac{4T}{m_e}\right)^{3/2} \exp(-\nu) \int_0^\infty \exp(-x) \sqrt{1-\frac{\gamma}{\gamma_l}\frac{x+\nu}{x+\nu_l}}\sqrt{x}\, dx, \tag{6}$$

where $\nu = U/2T$, $x = V/2T$, and the index e is related to electrons.

Let us consider low-energy ions (Fig.1, to the right). It is clear that ions with energies, for which the dependence $G^2(s)$ is given by the curve from the left of the discontinuity point of G^2 do not exist. The polar ions, i.e., low energy ions for which both mirror points are located on one side of the magnetic equatorial plane [10], when, and only when the considered point s lies between l and the mirror point, or

$$\gamma_n(x-\nu_n) = \gamma_l(x-\nu_l), \qquad \gamma_n \leqslant \gamma \leqslant \gamma_l, \tag{7}$$

i.e.,

$$x = \frac{\gamma_l\nu_l - \gamma_n\nu_n}{\gamma_l-\gamma_n}, \qquad 1 \leqslant \gamma_n \leqslant \gamma. \tag{8}$$

Therefore, to energy minimum corresponds $\gamma_n = \gamma$, and to energy maximum $\gamma_n = 1$,

$$x_{\min} = \frac{\gamma_l\nu_l - \gamma\nu}{\gamma_l - \gamma}, \qquad x_{\max} = \frac{\gamma_l\nu_l}{\gamma_l - 1}. \tag{9}$$

If the maximum of the function G^2 is to the left from the considered point s, the integration by pitch-angles is performed within the limits $(\arcsin\sqrt{G_n^2/G^2}, \pi/2)$; in the opposite case it is performed within the limits $(\arcsin\sqrt{G_l^2/G^2}, \pi/2)$. However, $G_n^2 = G_l^2$, so that in both cases the integral is written in an identical fashion. Inasmuch as the lower limit of integration by energies for polar ions coincide, while after intergation by pitch-angles the integrands are found to be identical, the density of ions has finally the form

$$n_i = 2\pi f_{0i}\left(\frac{4T}{m_i}\right)^{3/2}\exp(\nu)\int\limits_{(\gamma_l\nu_l-\gamma\nu)/(\gamma_l-\gamma)}^{\infty}\exp(-x)\sqrt{1-\frac{\gamma(x-\nu)}{\gamma_l(x-\nu_l)}}\sqrt{x}\,dx. \tag{10}$$

Effecting the density normalization of electrons and ions in the equatorial plane, we shall write the quasineutrality equation in the form

$$\begin{aligned}
&\exp(\nu)\int\limits_0^{\infty}\exp(-x)\sqrt{1-\frac{1}{\gamma_l}\frac{x}{x+\nu_l}}\sqrt{x}\,dx \times \\
&\times \int\limits_{(\gamma_l\nu_l-\gamma\nu)/(\gamma_l-\gamma)}^{\infty}\exp(-x)\sqrt{1-\frac{\gamma}{\gamma_l}\frac{x-\nu}{x-\nu_l}}\sqrt{x}\,dx = \\
&= \exp(-\nu)\int\limits_{\gamma_l\nu_l/(\gamma_l-1)}^{\infty}\exp(-x)\sqrt{1-\frac{1}{\gamma_l}\frac{x}{x-\nu_l}}\sqrt{x}\,dx \times \\
&\times \int\limits_0\exp(-x)\sqrt{1-\frac{\gamma}{\gamma_l}\frac{x+\nu}{x+\nu_l}}\sqrt{x}\,dx.
\end{aligned} \tag{11}$$

RESULTS OF NUMERICAL CALCULATIONS

It follows from formulas (6) and (10) that in the absence of a longitudinal field the density of particles is proportional to $\sqrt{1-\gamma/\gamma_l}$, and the quasineutrality equation (11) is transformed into an identity.

Effecting the substitution of the integration variable in the integral yielding the density of ions,

$$x = u + (\gamma_l\nu_l - \gamma\nu)/(\gamma_l - \gamma)$$

and introducing the denotation

$$R(a,b) = \int\limits_0^{\infty}\exp(-x)\sqrt{\frac{x+a}{x+b}}\sqrt{x}\,dx,$$

we shall represent the quasineutrality equation in the following form

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$$\exp\left(\nu - \frac{\gamma_l\nu_l - \gamma\nu}{\gamma_l - \gamma}\right) R\left(\frac{\gamma_l\nu_l}{\gamma_l - 1}, \nu_l\right) R\left(\frac{\gamma_l\nu_l - \gamma\nu}{\gamma_l - \gamma}, \frac{\gamma(\nu_l - \nu)}{\gamma_l - \gamma}\right) = \exp\left(-\nu - \frac{\gamma_l\nu_l}{\gamma_l - 1}\right) R\left(\frac{\gamma_l\nu_l}{\gamma_l - 1}, \frac{\nu_l}{\gamma_l - 1}\right) R\left(\frac{\gamma_l\nu_l - \gamma\nu}{\gamma_l - \gamma}, \nu_l\right), \quad (12)$$

The algebraic Eq.(12) was resolved with the aid of a computer using standard sub-programs for finding the root of the function over the given segment and computing the integral by the Simpson method. The results of computation of potential and density distribution of charged particles along the magnetic line of force as a function of γ is plotted in Fig.2. For the calculations we assumed the value γ_l = 200, which corresponds, for example, to the position of the absorption plane for the field line with L = 8 at the distance of 1.1 R_E.

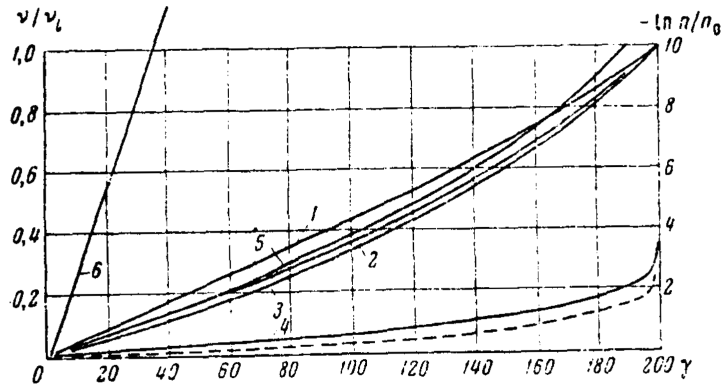

Fig.2

A series of curves is plotted for ν_l = 1, 10 and 100. Curves 1, 2, 3 give the distribution of the dimensionless potential ν/ν_l, and curves 4, 5, 6 illustrate the distribution of the natural logarithm of charged particles' density ratio at the equator, n_0 to that at the given point, n. The dashed curve indicates the distribution of the logarithm of this ratio of densities in the absence of a longitudinal electric field. At the absorption plane the density of charged particles is zero. It may be seen from Fig.2 that the distribution of the potential is little dependent on the quantity ν_l, i.e., on the ratio of potential difference between the equator and the absorption plane to plasma temperature. The corresponding dependence between the potential and γ is near a direct proportionality, i. e., roughly speaking, the electric field is proportional to $\text{grad}_{\parallel}B$, as would be in the case of monoenergetic distribution [9]. The difference consists in that for greater γ the proportionality factor is greater by a factor of 2 - 3 than for $\gamma \approx 1$, whereupon this difference increases somewhat with the rise of ν_l The value of ν_l is more substantially manifest in the distribution of charged particle density along the field line, which is seen from the comparison of the dashed curve with curves 4, 5, 6.

Attention should be drawn to ion cutoff from the side of low energies. It stems from expression (10) that at the considered point there are no ions with energy x lower than $(\gamma_l\nu_l - \gamma\nu)/(\gamma_l - \gamma)$. Indeed, during their motion in the direction toward the point $\gamma = \gamma_l$ the ions are accelerated by the electric field, and if the energy x is sufficiently low, the longitudinal energy rises more rapidly than the transverse energy, the pitch-angle does not attain $\pi/2$ at the point y_l so that particles depart from the considered system. When the electric field has an inverse direction, the same takes place with electrons.

In the Earth's magnetosphere such a tube of magnetic lines of force is not unidimensional. Particles drift along the magnetic lines of force in the azimuthal direction and move into the depth of the magnetosphere. At the same time they hit the regions where the direction of the longitudinal electric field reverses. This is why cutoff takes place for both ions and electrons from the side of low energies, as particles of low energy are accelerated by the longitudinal electric field and pour out into the atmosphere in polar regions. Inasmuch as inside the magnetosphere the plasma is in a state of motion [1, 11], while the magnetic field is frozen-in, the density of the matter is a function of magnetic field intensity. Consequently, the required matter density in the inner regions of the magnetosphere must be created at the expense of particles of sufficiently high energies; the lower limit of energy is of the order of the longitudinal difference in the potentials. At the present time the verification and the processing of this hypothesis on the formation mechanism of the soft component in the outer belt of trapped radiation is difficult [12], since theoretical, as well as experimental data on the distribution of the electric field in the Earth's magnetosphere are not available.

Thus, in one-dimensional tube of magnetic lines of force, bounded by a plane above which particles are fully abosrbed, the longitudinal potential difference at quasi-Maxwellian distribution function is approximately proportional to the intensity of the magnetoc field; the value of the potential difference ratio to temperature of the plasma has little influence on the distribution of the potential, and is essentially manifest in the density distribution of charged particles. If the electric field is directed from the equator, low energy ions are absent, and in the opposite case electrons of low energies are absent. This allows us to venture the assumption on the formation mechanism of the soft component in the outer belt of trapped radiation, linked with the escape of low energy charged particles, accelerated by the longitudinal electric field.

**** T H E E N D ****

Received on 19 May 1967

CONTRACT No.NAS-5-12487
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Translated by

ANDRE L. BRICHANT on 31 Au. 1968.

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